World Down Syndrome Congress
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Developing mathematical understanding based on geometric intuition in children with trisomy 21

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Mathematical education for people with trisomy 21:
questioning two basic assumptions…

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- Prevalent utilitarian view of mathematics

Mathematical education for people with trisomy 21: questioning two basic assumptions…

- Prevalent utilitarian view of mathematics

- Mathematics is a hierarchical discipline

…with consequences…

In the goals of teaching → Numeracy

In the choice of contents → Arithmetical skills

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…that deprive them…

Of the oportunity to be introduced in
more formative areas of mathematics

…Despite some existing good ideas

"Study mathematics for **its own sake**"
(Faragher & Clarke, 2014)

"Mathematics can be learned by people with an intellectual impairment **in a parallel way** …
because they seem to learn each part at a different rate"
(Monari &Benedetti, 2011)

We propose…

Numeri e forme (2016)

To base on **geometry** to understand mathematics

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Recovering XIX century innovators´optimism

Edouard Séguin (1812-1880)

Epistemological Reasons

Continuum intuition (René Thom, 1971)

Common roots of geometry and arithmetic (Lafforgue, 2010)

Cognitive profile of people with trisomy 21

Visual strenght (Bird&Buckley, 2012)

Interest in abstract symbols (Zimpel, 2016)

We propose…

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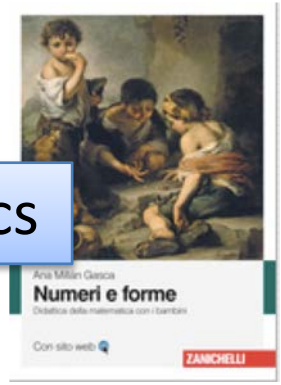

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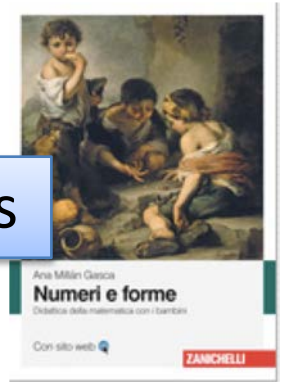

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- Geometry helps children to embody abstract ideas from direct experience

- Geometry has a crucial role in undersatanding the concept of number

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Geometry helps children with Trisomy 21
achieve higher levels of mathematical understanding
and thinking

Proyecto

TRISOMÍA 21

Matemáticas y Pensamiento

Una perspectiva abierta a todos

An open approach

Our convictions

Optimism about their learning competence

How are going children with Down syndrome to learn some mathematics if we **give them** *less* ***stimuli*** *than to non-disabled ones?*

(Monari, 1995)

High expectations

We cannot be sure what the child takes from mathematics classroom experiences but we can be sure that if the teacher restricts the task, language used, challenge or choice of approaches this inmediately excludes the child and limits ***opportunities for the child to learn as much as they are able***

(Clarke &Faraguer, 2016)

Didactical keys

- Solid mathematical basis
- Variety of activities
- Mimesis as a didactical tool

Didactical keys

Solid mathematical basis

Axiomatic of Hilbert for Geometry

Primitive concepts

Point
Straight line
Plane

Axioms

1. Conexion (**to pass through**)
2. Order (**to be between**, **segment**, **angle**)
3. Congruence (**comparison**)
4. Continuity (**measurement**)

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Related with primordial concepts

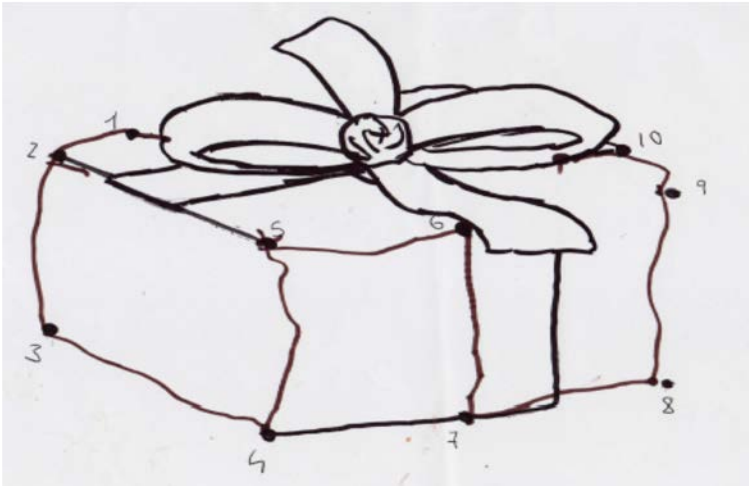
Beginning to teach from the simplest issues
Use the conceptual net
(Lafforgue, 2007)
Understand concepts through their relationships

Point

Stars

Stars in a constellation

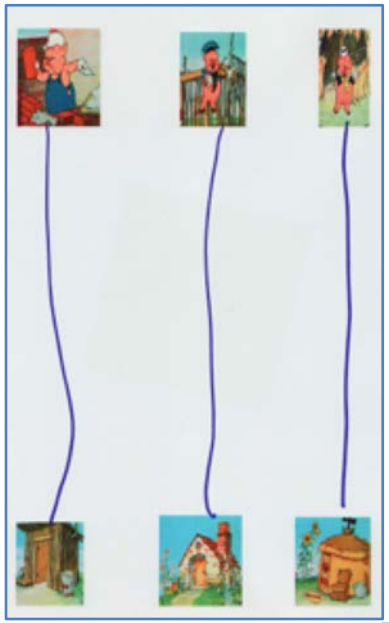
Points to count



Points in solids

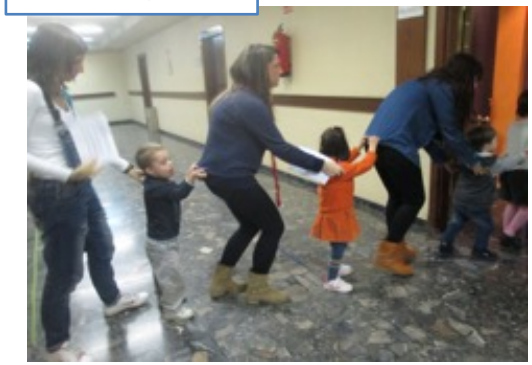
Straight line

Joining two points

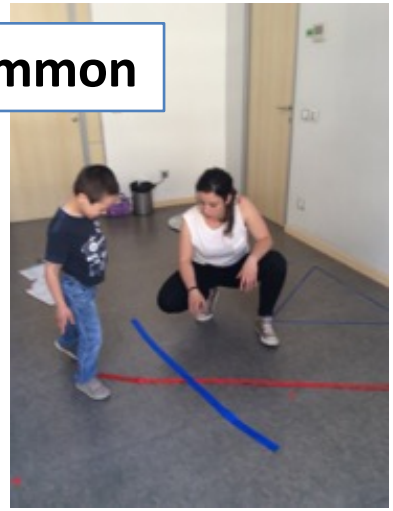


Going ahead

Lining up



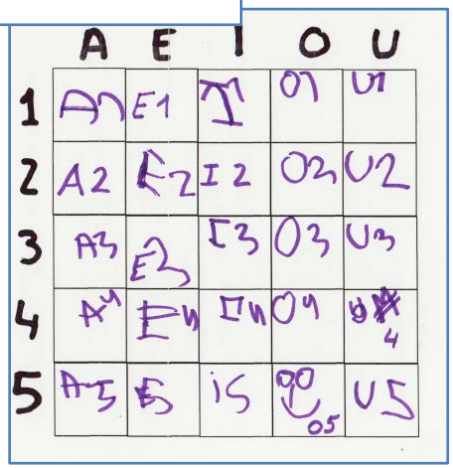
Point in common



Longer and longer…

Plane

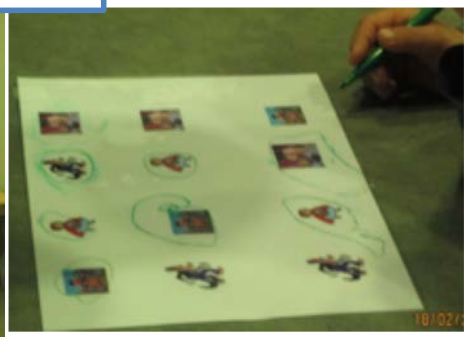
Let´s organize the plane…x



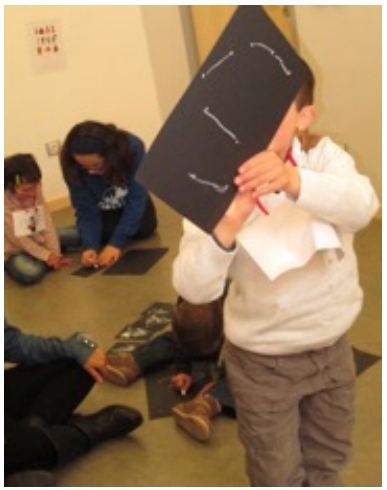
…to know it better!

Betweenness

What does it mean *to be between?*



Drawing segments…



…marking their points

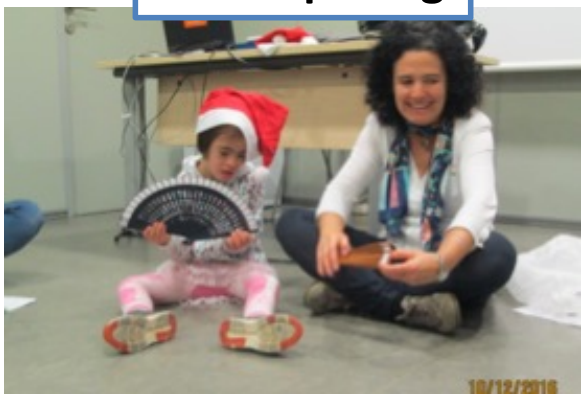
Numbers *are between*

Angles

“Opening” angles

Less than a right angle

Wide opening



From the “opening” to the plane

Portion of plane

Didactical keys

Variety of activities

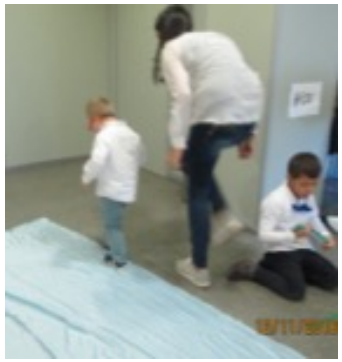
Didactical keys

Variety of activities

- Working in the **representative space** (Poincaré, 1902)

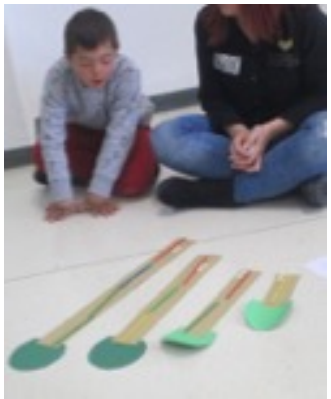
Movement

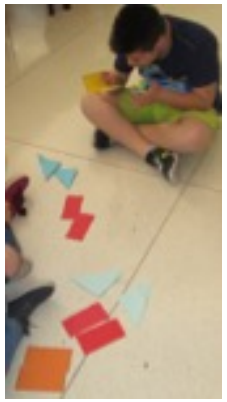
Rhythm

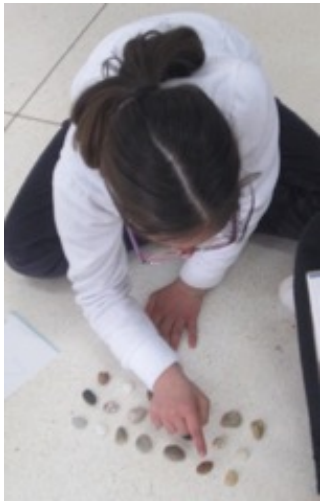


Without oral language

Using geometrically-inspired material





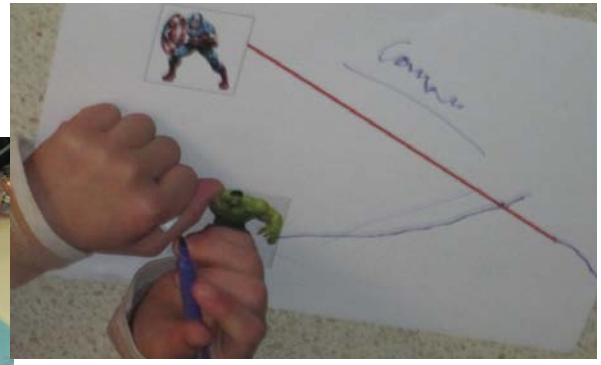


Didactical keys

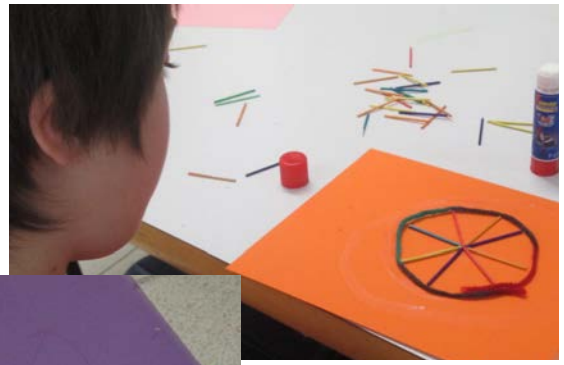
Variety of activities

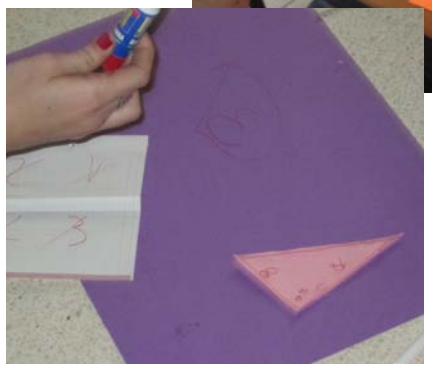
- Introducing them slowly in the **abstract geometric space**

Using paper and pencil



First levels of symbolic representation





Didactical keys

Mimesis as a didactical tool

Aristóteles, *(Poética)*
«as-similate»

Scaramuzzo, (2013)
Educazione poetica

For facing their difficulties about

Scarce expressive language

Motor Delay

Slow learning pace

Using one of their strengths

The power of personal imitation

A play: “as if we were…”

Body mimesis

Teacher as a model

Consists of…

- Design of activities according to the previous didactical keys

- Tested with children in an out-of-class context
  - Mathematical workshop for children with Trisomy 21
  - Study case
  - Experimental arithmetical focused programme

Consists of…

- A qualitative research ***based on practice***

  Phd: *Didactic of mathematics for children with Down syndrome from an integrated approach of elementary arithmetic and geometry*

  Line of research in University of Zaragoza: *Design of didactical mathematical sequences for children with special needs*

- Whose goal is to ***improve this practice***

  Teacher innovation project: *Learning to learn mathematics: what do children with intellectual disabilities teach us?*

General conclussions

Geometrical intuition of children with Trisomy 21 shows a great **potential**

- to favor the developing of mathematical ideas involving a certain abstraction
- To reinforce arithmetical contents
- to improve communication skills and abilities to think and learn
- to make the world more intelligible for them

General conclussions

Fertility of this approach that takes into account their strengths

Shifting the focus to geometry instead of arithmetic

Adequating methodologies to scape rote learning

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Transformative power of mathematics for facing biological conditions

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